Enhancing Prospective Mathematics Teachers' Pedagogical Knowledge of Mathematical Relations and Equivalence Relations

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***Abstract**: Fourth-year prospective mathematics teachers served as instructors and second-year future elementary school mathematics teachers participated as students in a unit on mathematical relations. Videotapes revealed that the fourth-year pre-service teachers were moderately successful in developing their second-year peers' understanding of the properties of a relation. The videos also showed that the instructors were prone to tell their students about relations without student interaction. Results indicated that participants valued the opportunity to work in a group to develop and improve a lesson, and the experience of observing, and in some cases teaching, a lesson that incorporated their joint efforts. Participants also reported the need to include more examples in the instructional unit and more class participation.*

1. Background

Lesson study, a collaborative professional development approach that originated in Japan, is gaining widespread popularity in mathematics education (see [1] and [2]). In the lesson study process, a group of teachers meet as a team to set goals and to carefully craft and collaborate on the design of a lesson. Once the lesson is designed one of the teachers in the group teaches the lesson, while the other group members observe the lesson. Later the whole group evaluates and reflects on the lesson discussing ways to reteach it as a means of making it more effective and stimulating for the students. The revised lesson is taught, observed, and reflected upon a second time.

Chokshi and Fernandez [3] believe that lesson study provides a large potential influence on the impact of the professional development of teachers as it creates a professional knowledge among teachers, while offering a connection between educational policy and practice. Fernandez [4] goes on to state that lesson study can provide opportunities for teachers to learn about an academic subject in a way that promotes the enactment of reform-minded teaching. Hiebert and Stigler [5] concur with this notion and describe lesson study as an institutionalized teaching improvement system built on research and the idea that teaching is a complex, cultural activity. Particularly, lesson study enables an activity structure that affords shareable and upscaleable knowledge for teacher professional development.

2. Aims of the Study

As part of their senior-level mathematics education seminar, twelve pre-service mathematics teachers reviewed research literature on the lesson study process in preparation for planning and implementing of an actual lesson study. Prior to preparing an instructional unit on mathematical relations, the twelve future teachers prepared discussion papers that outlined their goals, objectives, and expectations for the lesson study. These papers allowed the future teachers to express their personal concerns before undertaking the lesson study.

Within these papers, the pre-service teachers discussed their desire to work well within each study group, to involve students in the instructional activities, to enable students to clearly understand the instructional content, and to enable the future teachers to understand how to evaluate their own and other's teaching. In the discussion papers the future teachers also expressed reservations about only being able to observe the lessons on videotape instead of being present at the lesson and the time-consuming nature of the lesson study process. The future teachers also wrote about their expectations of viewing different types of effective teacher behavior and of their anticipation of using the lesson study process to learn how to adapt and revise a lesson before re-teaching it. In addition, the future teachers who were scheduled to teach the first lesson related their nervousness in teaching a topic that they did not feel they fully grasped, while the future teachers who were scheduled to teach a second (a revision of the first) lesson wrote of their hopes to improve the quality and presentation of the lesson and to use the lesson study experience to make themselves better prepared teachers.

3. Methodology

This study involved two tiers of participants. The first tier was comprised of twelve pre-service mathematics teachers who were enrolled in a senior-level mathematics education seminar planned and implemented the lesson study. Five of these future teachers jointly prepared a lesson plan on relations and equivalent relations that included the lesson's relation to NCTM Standards, teaching procedures, worksheets, and assessment. The second tier included twenty-five students who were enrolled in a second course of mathematics for elementary school teachers and served as the subjects of the study. These students were further divided into two groups (Group 1 and Group 2).

Two of the pre-service teachers then taught the lesson to a group (Group 1) of prospective elementary school teachers. The lesson was videotaped and later viewed by all twelve of the pre-service mathematics teachers who wrote reaction papers to the lesson. During a subsequent class the future teachers discussed ways to improve the lesson based on results from the in-class worksheets, assessments, and from debriefings by the two participants who taught lesson.

Two other seminar participants, neither of whom had written the first lesson plan nor taught the first lesson, prepared a second, revised lesson plan and taught the lesson to another group (Group 2) of prospective elementary school teachers. After the videotaping of the second class, all twelve future teachers viewed the tape and wrote reaction papers. The videotape of the class instruction and the pre-service teachers' reactions were discussed during the next seminar meeting. The prospective

teachers were then instructed to write a summary paper of their overall views on lesson study and on their lesson study experiences.

4. Instructional Materials for Lesson One

The instructional materials used in the first lesson included an overhead transparency that described a relation on a set X as "any set of ordered pairs in which the first and second components are from X." Reflexive, symmetric, and transitive properties of a relation were also described for a relation on a set X, and an equivalence relation was described as any relation on X that satisfies these three properties. The lesson also included three tasks on activity sheets for the students to complete.

During the first lesson students were given the definition of a relation and an explanation of its properties. They were then given an example which asked them to indicate whether the "relations on the set of all people are reflexive, symmetric, or transitive" and instructed to complete the following table.

Table 1 Table of Relations and Properties of Relations from the First Activity Sheet

Relation	Reflexive	Symmetric	Transitive
"is an ancestor of"			
"is a different age than"			
"has the same income as"			
"knows a telephone number for"			

The second activity sheet asked the students to determine if the relations given by three arrow diagrams were reflexive, symmetric, or transitive as in the next diagram.

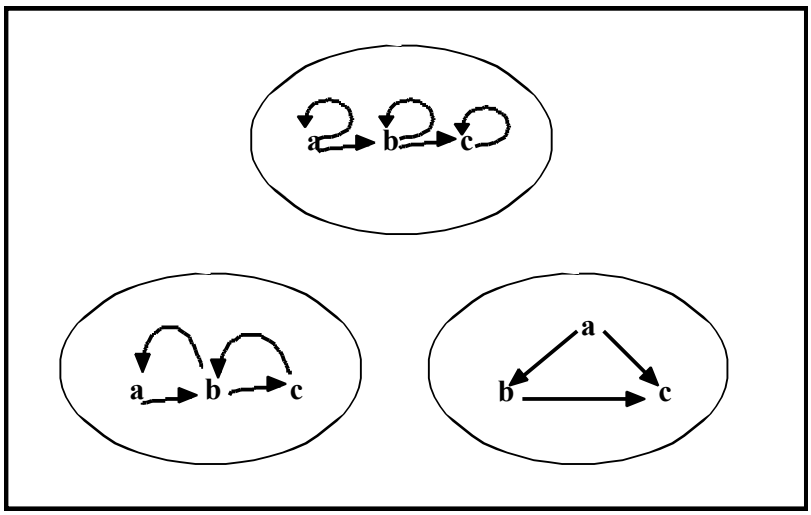

Figure 1 Diagrams of Relations from the Second Activity Sheet

The third activity sheet asked the students to draw ovals and construct arrow diagrams that showed relations that were: (1) reflexive and symmetric, but not transitive; (2) reflexive and transitive, but not symmetric; and (3) symmetric and transitive, but not reflexive.

5. Instructional Materials for Lesson Two

The refined second lesson plan also included definitions of the reflexive, symmetric, and transitive properties as well as a definition of an equivalence relation. However, a few changes were made based on discussions during the seminar class. The second lesson plan included a paper copy of a transparency sheet containing a definition of a relation along with four examples of relations: "has the same decimal value as", "is bigger than", "$x^2 + y^2 = 1$", and "$x + y < 1$". For the relation "has the same decimal value as", an ordered pair member of the relation was presented with the statement "(1 / 2, 2 / 4) is a relation". For the relation "is bigger than", the example was the statement "5 is related to 3 but 3 is not related to 5". For the relation "$x^2 + y^2 = 1$", the example included the statement that "(1, 0) is a relation, but (2, 2) is not". For the relation "$x + y < 1$", the example contained the statement "(-1, 0) is a relation, but (2, 5) is not". A table similar to the one given to the first group was used except that the relation "is taller than" replaced "is an ancestor of" and the relation "knows a phone number for" replaced "knows a telephone number for". Also included with the second lesson plan were the same second and third activity sheets that dealt with arrow diagrams.

6. Results

After both Group 1 and Group 2 students had completed the lesson on relations, they completed test items that used arrow illustrations to represent the reflexive, symmetric, and transitive properties. The test items particularly asked students to illustrate situations that were 1) symmetric and transitive, but not reflexive; 2) reflexive and symmetric, but not transitive; and 3) reflexive and transitive, but not symmetric. Results showed that students responded with illustrations that were arrow diagrams.

In Group 1, 60% of the students illustrated a situation that was symmetric and transitive, but not reflexive; while only 3 of the 6 correct illustrations depicted the transitive property for a relation on a set of 3 elements. Also, 30% of Group 1 students illustrated a situation that was reflexive and symmetric, but not transitive while only one Group 1 student illustrated a situation that was reflexive and transitive, but not symmetric.

In Group 2, 30% of the students illustrated a situation that was symmetric and transitive, but not reflexive, and each of the 3 correct illustrations depicted the transitive property for a relation on a set of 3 elements. Half of the Group 2 students correctly illustrated a mathematical situation that satisfied the reflexive and symmetric properties, but not the transitive property. Forty percent of Group 2 students illustrated a situation that was reflexive and transitive, but not symmetric.

7. Students' Reactions to the Lessons

Students from both Groups 1 and 2 were asked to write their reactions to the lesson on relations, their properties, and equivalence relations. In particular, they were asked to comment on what aspects of the lesson had been most helpful to them and to give an overall assessment of the lesson. When asked which aspect was most helpful, 40% of the participants from Group 1 responded

favorably to the diagrams; whole 20% noted the examples and 10% responded each to the explanations and the overhead transparency. Group 2 also responded favorably to the diagrams (40%); followed by 17% for examples and 8% for the explanations. No participants in Group 2 rated the overhead transparency as being helpful. Notably 70% of the students from both Group 1 and Group 2 rated the class presentations as being very good.

8. Seminar Participants' Overall Comments and Discussion of Their Experiences with the Lesson Study

At the conclusion of the viewing of videotape of the second lesson, the pre-service mathematics teachers were asked to prepare a final paper describing their experiences with the lesson study and any overall comments they had about the study. A review of the final papers showed that the participants valued:

(1) Opportunities to experience the lesson study process within the setting of an academic classroom,

(2) Experiences of working in a group to develop and improve a lesson, and

(3) Opportunities to observe, and in some cases teach, a lesson that incorporated the joint efforts of the seminar participants.

The participants also reported that they had added to their repertoire of teaching skills by observing and discussing the videos of the lesson. A majority of the seminar participants also reported that they saw improvements in the lesson implementations in moving from the first to the second lesson.

On the other hand, seminar participants expressed some disappointment that their lesson did not engender more classroom participation and student involvement. They also noted the criticism levied by some participants about the preparation and classroom performance of participants who taught or prepared the plans for the first or second lesson. Nearly half the seminar participants made reference to examples that were presented in the first lesson as being confusing and in need of being reviewed for clarity before the lesson. A major thrust of the suggestions for improvement was for pre-typed overhead transparencies with the definitions, for more explanations and student involvement, and for more examples of relations and their properties.

Overall, most participants indicated that they would include more examples in the lesson, including more examples of equivalence relations, more mathematical examples, and more examples generated and constructed by students in the classroom. Increasing class participation was also a theme of the comments from most of the seminar participants.

9. Conclusion

According to Lewis, Perry, and Hurd [6], "Lesson study is not just about improving a single lesson. It's about building pathways for ongoing improvement of instruction (p.18)". This was exactly the situation that these pre-service mathematics teachers experienced in the context of mathematical

relations and equivalence relations. They developed working relationships within their lesson study groups that allowed them the opportunity to collaborate and reflect on the pedagogy underlying these types of relations. The prospective teachers showed evidence of enhancing their pedagogical knowledge by not only following a cohesive lesson study cycle that emphasized mathematical knowledge, but also by exhibiting affective knowledge of collegial qualities that supported learning.

Another benefit of the pre-service teachers' work on the mathematical and equivalence relations was their involvement in collaborative tasks that helped them systematically develop their understanding of recent educational reform. The topic of educational reform constituted a sizable component of the future teachers' seminar class, but the reality of implementing classroom reform became concrete through their involvement with the lessons on mathematical relations. Analysis of the videotapes and written lessons showed that the participants who taught the lessons were more prone to tell their students about relations without providing tasks for the students to conjecture and construct their own relations. During the seminar participants' commentaries on the video of the second lesson, it was apparent that most had come to recognize the need to involve students more actively in the construction of relations through experimentation and mathematical reasoning.

The future teachers' implementation of the lesson study on mathematical relations was not without challenges. In particular, some of the future teachers expressed anxieties about being videotaped, about making public their teaching and their own knowledge of relations, as well as about agreeing on common pedagogical grounds for joint lesson planning. They also experienced some of the challenges that relate to general pedagogical research skills such as posing researchable questions, designing classroom experiments, specifying types of evidence for collection, and interpreting and generalizing results.

Since lesson study is explicit in virtually all areas of the curriculum in Japan and elsewhere, it is certainly feasible to extend the use of the components used in this study to other content in mathematics teacher education. Successful extensions to other content are apt to require less instructional time and effort by selecting topics, like mathematical relations, that are relatively straightforward for prospective teachers to teach as demonstration lessons.

Hiebert and Stigler [5] concluded their review of lesson study in mathematics by noting that lesson study's chances of success in the U.S. were closely tied to society's willingness to recalibrate its expectations for change by adopting a long-term improvement strategy that guarantees more effective teaching, not 1 year in the future, but 20 years in the future. The results of this study of videotaped lessons on mathematical relations and equivalence relations with pre-service teachers bear out this long-range view.

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